A CAUCHY PROBLEM FOR A NONLINEAR HEAT CONDUCTION EQUATION ENCOUNTERED IN COMBUSTION THEORY AND GASDYNAMICS

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We derive in explicit form a solution of a nonlinear heat conduction equation from combustion theory, taking into account a standard equation of gasdynamics. Estimates of errors are made for the desired expansions, thereby making it possible to estimate the accuracy of the calculations.

Many heat conduction problems may be reduced to the solution of nonlinear ordinary differential equations, solvable by the methods considered in [1], which includes a bibliography relating to this question.

It is of interest to study the class of problems reducible to nonlinear equations of parabolic type with a divergent principal part, arising in various physical situations [2]. In particular, the class of equations introduced here, besides the application pointed out in this paper, is encountered in problems of magnetogasdynamics, chemical kinetics, ignition problems, etc. We base consideration of the problems of the type indicated on a study of the smoothness of the initial data of the problem.

We assume that in addition to heat conduction there will be an increase in the amount of material, the speed of this material at a given point and a given instant of time being dependent on the existing density. The coordinate x will be reckoned along the normal to the combustion front. We consider the equation

$$\alpha_1(u,\ x)\frac{\partial u}{\partial \tau}=\frac{\partial}{\partial x}[\Psi(u,\ \omega)]+\Phi_1(x,\ u,\ \omega)+\Phi_2(x,\ u) \tag{1}$$

where we assume that the functions α_1; Ψ; Φ_1, and Φ_2 are defined for all values of u and (τ, x) of Π_T, where $\Pi_T = \{(\tau, x)\colon 0 \le \tau \le T,\ x \in E_n\}$, $\omega = \partial u/\partial x$. We say that the solution $u(\tau, x)$, defined for $\tau \in [0, T]$, lies in Π_T if, as a function of x, it belongs to Π_T for each fixed τ of $[0, T]$. In addition, we assume that in Π_T, except for some closed set containing at least one point with known coordinates, the coefficients in Eq. (1) satisfy conditions of analyticity.

1. If we consider Eq. (1) in the gasdynamic approximation, i.e., when $\Phi_2 = 0$, then the presence in the equation of terms involving derivatives of the second order corresponds to taking account of viscosity and heat conduction of the gas [3, 4]. As is evident from Eq. (1), the class of viscosities may be considered much broader than the class of linear viscosities, where there is not dependence of the limiting solution on the concrete form of the viscosity, i.e., we consider a broad class of divergent viscosities which does not, however, violate continuous dependence on the initial data. Thus for our problem we obtained a nonlinear viscosity for a broad class of functions $\Psi(u, \omega)$ [3, 4]. In view of the smoothness conditions imposed, the divergent form of Eq. (1) can, with no loss of generality, be a different one. This is not the case for discontinuous solutions [3]. Therefore in the class of sufficiently smooth functions the domain of definition of the functions $\Psi(u, \omega)$ may be widened.

2. Assume now that $\Phi_1(x, u, \omega) = 0$ and that $\Phi_2(x, u) \ne 0$; then Eq. (1) will correspond to the normal flame propagation process, where Φ_2 represents the speed of the reaction multiplied by the thermal effect of the reaction mentioned above.

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We note that in the case $\Phi_1(x, u, \omega) = 0$ and also in the case $\Phi_2(x, u) = 0$ the equation obtained from Eq. (1) is still not sufficiently accessible for study. Equation (1) includes as special cases the nonlinear equation of normal flame propagation and also the nonlinear equation of gasdynamics with a divergent viscosity.

We consider Eq. (1) subject to the initial condition

$$u(x, \tau)|_{\tau=0} = u_0(x);\ x = (x_1, \ldots, x_n) \in E_n. \tag{2}$$

We assume that Ψ; Φ_1; $\Phi_2 > 0$. We assign to these functions the form most important to the applications:

$$\Psi(u, \omega) = \gamma(u)\frac{\partial u}{\partial x};\ \Phi_1(x, u, \omega) = \beta(x)\frac{\partial \varphi(u)}{\partial x}. \tag{3}$$

As can be seen, the right side of the second of Eqs. (3) has a divergent form since it is obtained from the standard equation of gasdynamics upon assuming that $A(u) = d\varphi(u)$, i.e., that $A(u)du$ is the total differential of some vector function $\varphi(u)$. Here $A(u)$ is the coefficient of the first derivative. Let $\varphi(u)$ be a sufficiently smooth function of the argument u at each value u of Π_T. When $\varphi(u)$ is an arbitrary nonconvex function, the associated difficulties have not as yet been completely resolved. We assume therefore that $\partial^2\varphi/\partial u^2$ changes sign at a finite number of points. For $\varphi(u)$ we choose an arbitrary polynomial of the fourth degree. This enables us, with a high degree of accuracy, to account for nonlinearities of the unknown function up to the fourth order inclusive, i.e.,

$$\varphi(u) = \sum_{n=0}^{4} b_n u^n. \tag{4}$$

Solely for the sake of convenience, in the function $\Phi_2(x, u)$ we explicitly separate the thermal function δ, varying from point to point; we let

$$\Phi_2(x, u) = \delta(x)\Phi^*(u). \tag{5}$$

In Eq. (5) the function $\Phi^*(u)$ is considered to be defined in $(0, \infty)$, positive and monotonically increasing in this interval, which corresponds to the case of a large heat source, a characteristic property of a reaction; in particular, we assume that $\Phi^*(u) = u^r$, where r is an arbitrary positive number.

Taking into account Eqs. (3) and (5) and the properties of the function $\varphi(u)$, for Eq. (1) we obtain

$$\alpha(u)\beta(x)\frac{\partial u}{\partial \tau} = \frac{\partial}{\partial x}\left[\gamma(u)\frac{\partial u}{\partial x}\right] + \beta(x)\frac{\partial \varphi(u)}{\partial x} + \delta(x)\Phi^*(u). \tag{6}$$

Let G^* be a complex domain. We define $u_0(x)$ with the aid of an analytical transition in the domain G^* in such a way that in G^* we can display at least two sequences $\{y_n'\}$, $y_n' \to \infty$ and $\{y_n''\}$, $y_n'' \to \infty$ such that $\lim_{n\to\infty} \varphi(y_n') \neq \lim_{n\to\infty} \varphi(y_n'')$. When we consider all sequences $\{y_n\}$, $y_n \to \infty$ for which $\{\varphi(y_n)\}$ has a limit, then in accord with the general theory, we obtain

$$\lim_{y\to\infty} \frac{\ln \ln M^*_{u_0}(y)}{\ln y} = \Delta,\ M^*_{u_0}(y) = \max_{|u_0|=y} |u_0(x)|. \tag{7}$$

We take $\Delta = 2$. Then the solution of the Cauchy problem for Eqs. (1) and (6) in the indicated formulation exists and may be determined in a unique way for all the points of Π_T.

On the basis of the smoothness conditions imposed on the coefficients of Eqs. (1) and (6), and the conditions (7), we construct an explicit solution of the Cauchy problem

$$u(x, \tau) = \sum_{n=0}^{\infty} a_n(\tau)(x - x_0)^n \tag{8}$$

for the case in which $\partial^2\varphi/\partial u^2$ is of variable sign in the domain considered for variation of the arguments. Here x_0 is the point at which the combustion process is initiated.

Let ρ_n and c_n be the coefficients in the power series expansions of $\beta(x)$ and $\delta(x)$, respectively. Let $f_n^*(x, \tau)$ and $f^{*\prime}_{n,\tau}(x, \tau)$ denote the general terms of the sequences $\{a_n(\tau)(x-x_0)^n\}_0^\infty$ and $\{D_n(\tau)(x-x_0)^n\}_0^\infty$ with nonnegative integral indices. Suppose that, for each $\varepsilon > 0$, a number $N > N_0 > 0$ can be found such that for $N_k > N$ and $N^* > N$ the inequality

$$\left|\sum_0^{N^*} a_n(\tau)(x - x_0)^n - \sum_0^{N_k} a_n(\tau)(x - x_0)^n\right| = \left|\sum_{N_k}^{N^*} a_n(\tau)(x - x_0)^n\right| < \varepsilon \tag{9}$$

is satisfied. Assume, moreover, that for an arbitrary choice of the sequence of values $N_1, N_2, \ldots, N_k, \ldots \to \infty$ as $k \to \infty$, for the sequence of functions $\{\sum_{n=0}^{N_k} f^{*\prime}_{n,\tau}(x, \tau)\}$, with $\varepsilon > 0$ arbitrary and for a specific $L_k = L^*(\varepsilon)$, the inequality

$$\sup_{0 \leqslant \tau \leqslant T} \left| u(x, \tau) - \sum_{n=0}^{N_k} f^{*\prime}_{n,\tau}(x, \tau) \right| < \varepsilon \tag{10}$$

is satisfied. Inequality (10) must be satisfied for $N_k > L^*(\varepsilon)$ for all $\tau \in [0, T]$. Then dividing Eq. (6) by $\alpha(u)\beta(x)$, on the left side of the equation we will have

$$\alpha_0 \frac{\partial u}{\partial \tau} = \alpha_0 \sum_{n=0}^{\infty} D_n(\tau)(x - x_0)^n. \tag{11}$$

Assume that analogous conditions are satisfied for the sequence of derived functions $\sum_{n=0}^{N_k} f^{*\prime}_{n,x}(x, \tau)$ as $N_k \to \infty$. In this case, if $\{a_n(\tau)\}$ and $\{a_n(\tau)\}^{-1}$ are arbitrary sequences of those enumerated, then, subject to the condition that $\beta(x)$ belongs to the class $C\{n!\}$, we obtain the following expansion for the essential nonlinearity of Eqs. (1) and (6):

$$2\gamma_0 \frac{1}{u\beta(x)} \left(\frac{\partial u}{\partial x}\right)^2 = 2\gamma_0 \sum_{n=0}^{\infty} K_n^{(4)}(\tau)(x - x_0)^n, \tag{12}$$

where

$$K_n^{(4)}(\tau) = a_1(\tau)(n+1)a_{n+1}(\tau) + \sum_{i=1}^{n} i a_i(\tau) K_{n+1-i}^{(3)}(\tau);$$

$$K_0^{(4)}(\tau) = a_1^2(\tau); \; K_0^{(3)}(\tau) = a_1(\tau);$$

$$K_n^{(3)}(\tau) = (n+1)a_{n+1}(\tau) - \sum_{j=1}^{i} (n+1-j) a_{n+1-j}(\tau) \sum_{k=1}^{j} a_k(\tau) A_{j-k}^{(2)}(\tau);$$

$$A_0(\tau) = [a_0(\tau)\rho_n]^{-1}; \; A_n(\tau) = -A_0(\tau) \sum_{s=1}^{n} A_{n-i}^{(2)}(\tau) \sum_{p=0}^{s} a_p(\tau)\rho_{s-p}.$$

Analogous expansions are also obtained for the other terms in Eq. (6). We obtain, as a result, the following denumerable system of nonlinear differential equations:

$$\alpha_0 D_n(\tau) - \sum_{i=1}^{n} i a_i(\tau)[2\gamma_0 K_{n+1-i}^{(3)}(\tau) + a_{n+1-i}(\tau)] + (n+1)a_{n+1}(\tau)v(\tau)$$

$$- \sum_{i=1}^{n} W_{i,j}^{(1)}(\tau) - \frac{\gamma_0}{\rho_0} \sum_{i=1}^{n} (n+1-i)(n+2-i)a_{n+2-i}(\tau) \sum_{j=1}^{i} \rho_i F_{n-i}^{(1)}$$

$$+ \sum_{i=0}^{n} \eta_i \sum_{j=0}^{n-i} a_i(\tau) a_{n-i}(\tau) + \frac{\gamma_0}{\rho_0}(n+1)(n+2)a_{n+2}(\tau), \tag{13}$$

$$a_n(0) = \sigma_{sn} \quad (n = 0, 1, 2, 3, \ldots), \tag{14}$$

where

$$W_{i,j}^{(1)}(\tau) = \frac{2\gamma_0 a_2(\tau)}{\rho_0} \rho_i F_{n-i}^{(1)} + (n+1-i)a_{n+1-i}(\tau) \sum_{j=1}^{i} a_j(\tau) C_{i-j}^{(1)}(\tau);$$

$$C_0(\tau) = [a_0(\tau)]^{-1}; \; C_n(\tau) = -C_0(\tau) \sum_{s=1}^{n} a_s(\tau) C_{n-s}^{(1)}(\tau). \tag{15}$$

The structure of F_n coincides completely with that of Eq. (15), but involves terms of a polynomial which are constant and known. $K_{n+l-i}^{(3)}(\tau)$ is determined from the series of equations (12). In Eq. (14) the σ_{sn} are the coefficients in the expansion of $u_0(x)$.

We point out now an especially important particular case [3, 4] for the equations of gasdynamics. On a graph of $\varphi(u)$ let us fix two points, u^+ and u^-. For $u(x, \tau)$ we construct from $[u^+, u^-]$ a function $\zeta(u)$ from the class of functions satisfying the condition

$$\zeta(u_1', \ldots, u_n') - \zeta(u_1, \ldots, u_n) - (u_i' - u_i)\frac{\partial \zeta}{\partial u_i} < 0, \tag{16}$$

and coinciding with $\varphi(u)$ at the points $u = u^+$ and $u = u^-$, these functions being the smallest of all the functions which satisfy the condition $\zeta(u) \geq \varphi(u)$ on the interval $[u^+, u^-]$. It is obvious that in this case $\tan \alpha \neq 0$, where α is the angle of inclination to the u axis of the chord joining the points of intersection of the graphs of the functions $\varphi(u)$ and $\zeta(u)$. In this case the denumerable system of nonlinear equations is a special case of Eqs. (13)-(14) which we shall not write out here.

The series (8) is uniformly convergent by virtue of the restrictions (7) imposed on the order of growth of $u_0(x)$ and by virtue of the smoothness conditions imposed on the coefficients of Eqs. (1) and (6). We give an a priori estimate of the convergence of the series (8) for the case of a very weak nonlinearity of Eqs. (1) and (6), excluding from this the nonlinearity (12). Then upon extending the unknown terms into the domain G* and integrating them term by term [5], we obtain

$$\left| f_0^*(x - x_0, \tau) \right| \leqslant U^*(\tau) \exp\left[-\frac{\mu |x_1 - \xi_1|^q - \mu_1 |v_1 - w_1|^q}{\tau - t} \right] + U_0(\Lambda_0, C^*), \tag{17}$$

$$\left| f_n^*(x - x_0, \tau) \right| \leqslant U_n^*(\tau) \exp\left[-\mu \left(1 - \frac{n\varepsilon}{2+\varepsilon} \right) v \right] + \frac{\mu_1 (v_1 - w_1)^q}{\tau - t} + U_n(\Lambda_n, C^*), \tag{18}$$

where

$$U^*(\tau) = \frac{S}{\sqrt{(\tau - t)^{3-\varepsilon}}}; \quad U_n^*(\tau) = \frac{S\left(\dfrac{\varepsilon}{2+\varepsilon}\right)}{\sqrt{(\tau - t)^{3-n\varepsilon}}}.$$

Here $\mu = \mu(\varepsilon, \delta, T, \sup|s(x)|) = \text{const} > 0$; ε is the Hölder exponent for the coefficients in G_1; δ is a positive constant characterizing the parabolicity of the equation obtained after eliminating the nonlinearity (12):

$$C_1\{z = (z_1, x_2, \ldots, x_n);\ |z_1 - x_1^0| < v;\ -\infty < x < \infty,\ 0 \leqslant \tau_1 \leqslant \tau \leqslant \tau_2 \leqslant T\};$$

$$z_1 = x_1 + iv_1;\ \zeta_1 = \xi_1 + iw_1;$$

Λ_n is a correction on the nonlinearity; C^* is an upper bound to the analytic partial integrals obtained from the right side of Eq. (6),

$$S(x) = \sum_{n=0}^{\infty} F_n (x - x_0)^n.$$

Based on these estimates, we can, with the aid of the series (8), guarantee an arbitrary preassigned accuracy. To do this, we assign a value of the difference Λ^* between the left side and the right side of Eqs. (1) and (6) at the point x_0. We then make a check of the accuracy with respect to the equation as $x \to \infty$. Upon satisfying the condition $\Lambda^* > \delta^*$ ($\delta^* = \text{const} > 0$), we calculate new initial data at the point x_1, subsequent to which the whole process may be repeated. As the points x_i approach the singular point, Δx_i gradually diminishes and tends towards zero. The number of terms of the polynomial in the transition from point to point may vary. To improve convergence we can employ analytic continuation and, by analogy with [1], construct the sequence of series

$$u = \sum_{n=0}^{\infty} a_n(\tau)(x - x_0)^n;\ a_n(\tau) \equiv a_n^{(k)}(\tau);\ x_0 = x^{(k)}$$

with center at the points $x = x_0'$; $x = x_0''$; . . . , etc., which converge faster than the unknown series. Upon effecting the transition from point to point, the transition modulus [5] will satisfy

$$\frac{|\omega(x_0) - \omega(x_1)|}{\rho^*(x_0, x_1)^{\varepsilon}} \leqslant Z_0 \left(\sup|u| + \left| \frac{\partial}{\partial x} \Gamma\left(\gamma_0, x, \frac{\partial \omega}{\partial x}\right) + V(x, u, \omega, \gamma_0) \right|_{2+\varepsilon}^{\Pi[0,T]} \right). \tag{19}$$

From inequality (19) the modulus of continuity of derivatives of much higher order in $\Pi_{[0,T]}$ readily follows. Here

$$\Gamma\left(\gamma_0, x, \frac{\partial \omega}{\partial x}\right) = \frac{1}{\gamma_0 \beta(x)} \cdot \frac{\partial \omega}{\partial x};\ V(x, u, \omega, \gamma_0) = V_1(u, \omega, \gamma_0) + V_2(x, u, \omega);$$

$$V_1(u, \omega, \gamma_0) = 2\gamma_0 \frac{1}{u\beta(x)} \left(\frac{\partial u}{\partial x}\right)^2;$$

V_2 is a nonlinear function of its arguments.

As a consequence of the form of V_1, inequality (19) holds even with all the nonlinearities of Eqs. (1) and (6) present. It is obvious that inequalities (17)-(19) remain valid even for the functions $\zeta(u)$.

Let us write the system (13) in the form

$$\alpha_0 D_n(\tau) = \sum_{i=1}^{n} i a_i(\tau) K_i^{(5)}(\tau) + (n+1) a_{n+1}(\tau) v(\tau) - \sum_{i=1}^{n} W_{i,j}^{(1)}(\tau) - F_{i,j}^{(3)}(\tau) + \frac{\gamma_0}{\rho_0}(n+1)(n+2) a_{n+2}(\tau), \tag{20}$$

where

$$K_i^{(5)}(\tau) = 2\gamma_0 K_{n+1-i}^{(3)}(\tau) + a_{n+1-i}(\tau);$$

$$F_{i,j}^{(3)}(\tau) = \frac{\gamma_0}{\rho_0} \sum_{i=1}^{n-1} (n+1-i)(n+2-i) a_{n+2-i}(\tau) \sum_{j=1}^{i} \rho_i F_{n-i}^{(1)} - \sum_{i=0}^{n} \eta_i \sum_{j=0}^{n-i} a_i(\tau) a_{n-i}(\tau).$$

We consider, according to [6], the system of differential equations $H^*[a_n(\tau), \lambda^*]$, depending on a parameter and defined by the equation

$$H^*[a_n(\tau), 1] = \alpha_0 D_n(\tau) - \sum_{i=1}^{n} i a_i(\tau) K_i^{(5)}(\tau) - (n+1) a_{n+1}(\tau) v(\tau)$$

$$+ \sum_{i=1}^{n} W_{i,j}^{(1)}(\tau) + F_{i,j}^{(3)} - \frac{\gamma_0}{\rho_0}(n+1)(n+2) a_{n+2}(\tau). \tag{21}$$

The operator $H^*[a_n(\tau), 1]$ naturally includes the initial unknown system of functions

$$\sigma_{s1}, \sigma_{s2}, \ldots, \sigma_{sn}, \ldots, \quad \text{here} \quad 0 \leqslant \lambda^* \leqslant 1. \tag{22}$$

Assume now that for $\lambda^* = 0$ the system $H^*[a_n(\tau), \lambda^*]$ has the obvious solution $a_n^0(\tau)$. We seek a solution of the system

$$H^*[a_n(\tau), \lambda^*] = 0 \tag{23}$$

as a function $a_n(\tau, \lambda^*)$ of the parameter λ^*. In determining the initial approximation it is sufficient to find $a_n(\tau, \lambda^*)$ approximately as a function of λ^*. By virtue of the analyticity of the Cauchy problem for Eqs. (1) and (6), the differential operator, defining the system of differential equations $H^*[a_n(\tau), \lambda^*]$, will possess adequate smoothness, therefore $a_n(\tau, \lambda^*)$, as a function of the parameter λ^*, must satisfy the differential equation

$$\frac{dH^*[a_n(\tau, \lambda^*), \lambda^*]}{da_n(\tau, \lambda^*)} \cdot \frac{da_n(\tau, \lambda^*)}{d\lambda^*} + \frac{dH^*[a_n(\tau, \lambda^*), \lambda^*]}{d\lambda^*} = 0 \tag{24}$$

subject to the condition

$$a_n(\tau, 0) = a_n^0(\tau). \tag{25}$$

To construct an approximate solution $a_n(\tau, \lambda^*)$ as a function of the parameter λ^*, we use Euler's broken line method. Subdividing the domain of variation of λ^* into m parts by the points $\lambda_0^* = 0 < \lambda_1^* < \ldots < \lambda_m^* = 1$, we obtain initial approximations for initiating the iterational process

$$a_n(\tau, \lambda_{i+1}^*) = a_n(\lambda_i^*) - \left\{\frac{dH^*[a_n(\tau, \lambda_i^*)]}{da_n(\tau, \lambda^*)}; \lambda_i^*\right\}^{-1} \frac{dH^*[a_n(\tau, \lambda_i^*); \lambda_i^*]}{d\lambda^*} (\lambda_{i+1}^* - \lambda_i). \tag{26}$$

Since the element $a_n(\tau, \lambda^*)$ is close to the exact solution of the system (20), and consequently also to that of the system (13)-(14), we choose it for the initial approximation, denoting it by $a_{n,0}(\tau, \lambda_{i+1}^*)$, after which the iterational process converges rapidly. Further, from the system (20) we form a sequence of linear systems. We seek an approximate solution [6] for each system of this sequence. The approximate solutions of each linear system will determine successive approximations $a_{n,k}(\tau, \lambda_{i+1}^*)$, which will tend to the exact solution $a_n(\tau)$. Applying Saidel's method to solve each linear system, we obtain the desired iterational process from the sequence of linear systems:

$$a_{n,k+1}(\tau) = A^*\left\{a_{n,k}(\tau); \frac{d}{da_n(\tau)}\left[\alpha_0 D_{n,k}(\tau) - \sum_{i=1}^{n} i a_{i,k}(\tau) K_{i,k}^{(5)}(\tau) - (n+1) a_{n+1,k}(\tau) v_k(\tau)\right.\right.$$

$$\left. + \sum_{i=1}^{n} W_{i,j,k}^{(1)}(\tau) + F_{i,j,k}^{(3)}(\tau) - \frac{\gamma_0}{\rho_0}(n+1)(n+2) a_{n+2,k}(\tau)\right];$$

$$a_{n,k}(\tau)\frac{d}{da_n(\tau)}\left[\alpha_0 D_{n,k}(\tau)-\sum_{i=1}^{n} ia_{i,k}(\tau)K_{i,k}^{(5)}(\tau)-(n+1)a_{n+1,k}(\tau)v_k(\tau)\right.$$

$$\left.+\sum_{i=1}^{n} W_{i,j,k}^{(1)}(\tau)+F_{i,j,k}^{(3)}-\frac{\gamma_0}{\rho_0}(n+1)(n+2)a_{n+2,k}(\tau)\right]-\alpha_0 D_{n,k}(\tau) \tag{27}$$

$$\left.+\sum_{i=1}^{n} ia_{i,k}(\tau)K_{i,k}^{(5)}(\tau)+(n+1)a_{n+1,k}(\tau)v_k(\tau)-\sum_{i=1}^{n} W_{i,j,k}^{(1)}(\tau)-F_{i,j,k}^{(3)}(\tau)+\frac{\gamma_0}{\rho_0}(n+1)(n+2)a_{n+2,k}(\tau)\right\}.$$

In proceeding, we denote the expression appearing within the braces by $b^*(\tau)$.

Based on the smoothness of the functions defining the system, the process (27) converges more rapidly than a geometric process. Moreover it is obvious that for the desired iterational process

$$\|a_{n,k+1}(\tau)-a_n(\tau)\|\leqslant(q+\delta_n)\|a_{n,k}(\tau)-a_n(\tau)\|. \tag{28}$$

Taking relations (27) and (28) into account, we obtain finally [6]

$$\|a_{n,k+1}^{*}(\tau)-a_n(\tau)\|\leqslant\frac{q}{1-q}\|a_{n,k}(\tau)-a_n(\tau)\|+\frac{1}{1-q}\|a_{n,k+1}(\tau)-a_n(\tau)\|. \tag{29}$$

Here $a_n^*(\tau)$ are the exact solutions of the corresponding linearized systems. As is evident from the conditions placed on $u_0(x)$ and the coefficients of Eqs. (1) and (6), the operator A^*, which describes the iterational process (27), has an adequately smooth derivative at the point $a_n(\tau)$. This enables us to write down for the system (20) formulas for speeding up the convergence analogous to L. A. Lyusternik's formulas for linear systems. Namely, if we consider

$$a_n(\tau)\approx a_{n,k}(\tau)-\frac{[a_{n,k}(\tau)-a_{n,k-1}(\tau);\ a_{n,k}(\tau)-a_{n,k-1}(\tau)]}{[a_{n,k+1}(\tau)-2a_{n,k}(\tau)+a_{n,k-1}(\tau);\ a_{n,k}(\tau)-a_{n,k-1}(\tau)]}[a_{n,k+1}(\tau)-a_{n,k}(\tau)], \tag{30}$$

we obtain, with reference to the iterants $a_{n,k-1}(\tau)$; $a_{n,k}(\tau)$, and $a_{n,k+1}(\tau)$, a more accurate value of $\tilde{a}_n(\tau)$ than $a_{n,k+1}(\tau)$.

The question of the stability of the solution of the Cauchy problem for Eq. (1) for the function $\xi(u)$ and for $\Phi_2 = 0$ was studied in [3, 4]. For the case $\Phi_1 = 0$, this problem was the subject of many investigations, in particular [3, 7]. For the case when $\Phi_1 \neq 0$ and $\Phi_2 \neq 0$, simultaneously, the stabilization is equivalent to the existence of a function $w^*(x)$ for which the following conditions are satisfied:

$$|u(x,\ \tau)-w^*(x_n)|<\varepsilon,\quad \text{as only}\quad |x-x_n|<\delta, \tag{31}$$

where δ is independent of x_n in each finite domain of the space E_n. To show this, we differentiate Eqs. (1) and (6) successively with respect to x, taking relation (7) into account, and thus obtain the inequality (31).

From the statement above, $u(x, \tau)$ is defined for $\tau \in [0, T]$. A unique solution $u_1(x, \tau)$ may be derived which is defined for $\tau \in [0, T + \Delta^*]$, Δ^* and is such that $u(x, \tau) \equiv u_1(x, \tau)$ for $\tau \in [0, T]$. Then $u_1(x, \tau)$ is an extension of the solution $u(x, \tau)$ to the point $T + \Delta^*$. In order for the desired solution to be continuable in the natural norm it is necessary that for its derivatives of orders $p + 1, \ldots, q$ on the finite interval $[0, T]$, a fine constant $R(T)$ can be found so that for the derivatives of the solution the following condition is satisfied [5]

$$\left|\frac{\partial^r u(x,\ \tau)}{\partial x^r}\right|\leqslant R(T),\ p+1\leqslant|r|\leqslant q. \tag{32}$$

Taking into account the smootheness in our problem and successively applying the condition (32), we can extend $u(x, \tau)$ onto an arbitrarily preassigned interval.

We now consider the case of a domain where the point x_0 changes its position as the time varies [8, 9]. In this case the series (8) assumes the form

$$u(x,\ \tau)=\sum_{n=0}^{\infty} a_n(\tau)[x-x_0(\tau)]^n. \tag{33}$$

We speak of the series (33) as an "instantaneous" series since it becomes the series (8) if, beginning with some instant τ_1, the motion of the point x_0 is terminated. Then when the conditions (7) and (9)-(10) are imposed, the right side of the system (17) stays the same while the left side assumes the form [9]

$$\alpha_0 D_n^*(\tau) = \alpha_0 \left[D_n(\tau) - (n+1)\, a_{n+1}(\tau)\, \frac{dx_0(\tau)}{d\tau} \right]. \qquad (34)$$

In view of the fact that $x_0(\tau)$ is an arbitrary smooth function (except for $x_0(\tau) = \tau^{-1}$), the point x_0 may change its position in an arbitrary way. In a given case the inequality (31) shows that at each point at a distance of $x_i(\tau)$ from the plane, a definite value of the temperature is established in the course of time.

In this paper in carrying out the calculational algorithms we used the following numerical characteristics: $\gamma(u) = \gamma_0 u^m$, i.e., in the form of a rapidly changing function of temperature: $b_0 = 16$; $b_1 = 0$; $b_2 = 3/2$; $b_3 = -2/3$; $b_3 = 0.25$.

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